BEYOND MARKET EQUILIBRIUM: THE FUTURE OF ACTIVE INVESTING

Integrating Factors in Market Indexes and Active Portfolios

Dimitris Melas, Zoltan Nagy, Navneet Kumar, Peter Zangari

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EXECUTIVE SUMMARY

Asset owners use indexes as policy benchmarks and reference portfolios in their asset allocation. Index investors track cap-weighted indexes that seek to capture the market return. Active investors select securities and build portfolios that aim to outperform the market. All these types of investors may be able to benefit from incorporating factors into their process. More importantly, they may also be able to integrate factors without compromising other fundamentally important aspects of their strategies.

In this article, we review factor performance in global equity markets using coherent data and methodology and apply a new template to evaluate backtests for potential selection bias under multiple testing (SBuMT). We then propose a systematic process for integrating factor information into different investment strategies. We show that this process is consistent with the Black-Litterman framework and test it on a sample of market indexes and active equity portfolios. Integrating factors in indexes improved risk-adjusted performance while retaining high liquidity and capacity. Adding factors to active strategies enhanced information ratios while maintaining the portfolio characteristics and stock selection alpha of the original strategies.

Our analysis may have important implications for different types of investors. Asset owners may be able to tilt reference indexes towards rewarded factors without sacrificing market coverage and diversification. Index managers can track factor-tilted indexes as they remain investable and replicable. Finally, active managers may be able to incorporate factor information into their strategies to harvest factor premia while preserving their core investment process and the added value from fundamental security selection.

FOUNDATIONS OF FACTOR INVESTING

The theoretical foundations of factor investing can be traced back to pioneering academic research published several decades ago. Markowitz (1952) provided an analytically tractable definition of risk and established mean variance optimization as a formal method for building portfolios by trading off risk and return. Sharpe (1964) introduced the Capital Asset Pricing Model that elegantly captures the idea that the market is the most important common driver of portfolio performance. Ross (1976) extended the market model to include multiple factors that may exert common influences on asset prices and portfolio returns.

A substantial body of empirical research followed over the next four decades, aiming to establish the precise nature of the common factors affecting risk and return in different asset classes and markets. Many studies also proposed hypotheses explaining why some of these factors may be priced and therefore why assets and portfolios that emphasize these characteristics may earn positive excess returns. Potential explanations include systematic risk, behavioral bias, asymmetric information, and institutional constraints. In equities, eight factor groups have been documented through empirical research and have been used extensively in portfolio risk models and in active investment strategies: value, size, momentum, volatility, quality, yield, growth and liquidity.

Value and size were established early on as important common influences and potential sources of excess return, for example, Basu (1977), Banz (1981), Brown (1983) and Fama and French (1993). Jegadeesh and Titman (1993) documented the existence of cross-sectional momentum in U.S. equities while Carhart (1997) added momentum to the Fama and French three-factor model. Black (1972), Haugen and Baker (1991) and Frazzini and Pedersen (2014) documented the low-volatility effect and established volatility as an important equity factor and potential source of excess return.

Sloan (1996) showed that accounting accruals are negatively correlated with future stock returns while Novy-Marx (2013) found that high profitability companies earn higher returns despite having higher valuations. Profitability and earnings quality are often viewed as different dimensions of the quality factor. Other metrics used to quantify quality include financial leverage, earnings variability and asset growth (investment quality). Several studies found a link between dividend yield and subsequent stock performance, including Blume (1980), Fama and French (1988) and Arnott and Asness (2003). Growth is a fundamental input into all valuation models and has been investigated in a number of empirical studies, for example, Ofer (1975), Bauman and Dowen (1988) and Fama and French (2006). Finally, several studies document the link between Liquidity and the cross section of returns, including Amihud and Mendelson (1989), Amihud (2002) and Pastor and Stambaugh (2003).

Despite the large number of studies examining factors, one challenge in evaluating the results is lack of consistency in terms of data sources, definition of variables, portfolio construction methodology, geographical focus, etc. Exhibit 1 shows historical performance statistics for the eight equity factors documented in the literature using point-in-time data and a consistent methodology covering global equity markets over the period Dec. 31, 1994 to Feb. 28, 2018. We examine different variables that are commonly used to quantify equity factors. The precise definitions of the variables we study can be found in Morozov et al. (2015). Also, we estimate historical factor performance in three different and increasingly sophisticated settings that account for and help us understand the influence of all important performance drivers, including countries, industries and other factors.

In the first setting, we formed equally weighted quintile portfolios sorted on a particular factor and examined the performance of a monthly rebalanced strategy that goes long the top quintile and short the bottom quintile. This simple strategy reflects the returns associated with a specific factor but also includes other influences such as countries, industries and other style factors. In the second setting, we run univariate cross-sectional regressions of stock returns against stock exposures to a particular factor. The regressions include indicator variables for countries and industries. Effectively, through this process we estimated factor returns net of country and industry influences. Finally, in the third setting, we ran multivariate cross sectional regressions of stock returns against countries, industries and all style factors. This process isolates returns associated with a particular factor, net of country, industry and other factor effects.

Exhibit 1: Historical Performance of Global Equity Factors

Factors	Equal Weighted Quintile Portfolios					Long-Short Quintiles			Univariate Regression			Multivariate Regression		
	Q1	Q2	Q3	Q4	Q5	Ret	Vol	Sharpe	Ret	Vol	Sharpe	Ret	Vol	Sharpe
Value														
Book to Price	-2.09	-1.54	-0.67	0.38	3.93	6.01	13.1	0.46	1.70	3.55	0.48	1.84	1.58	1.16*
Earnings Yield	-3.72	-2.33	-0.46	1.45	5.06	8.78	13.3	0.66*	3.55	3.27	1.08*	2.37	2.04	1.16*
Long Term Reversal	-3.96	-1.50	0.11	1.13	4.22	8.18	13.2	0.62*	1.70	2.21	0.77*	1.31	1.47	0.89*
Size														
Size	1.22	2.26	-0.48	-1.64	-1.38	-2.60	12.2	-0.21	-0.15	2.66	-0.05	-0.06	2.46	-0.02
Midcap	0.90	0.66	0.72	-0.52	-1.77	-2.67	10.5	-0.25	0.68	2.06	0.33	0.15	1.39	0.11
Momentum														
Momentum	-7.05	-3.56	0.42	3.73	6.49	13.54	21.4	0.63*	3.21	5.73	0.56	3.94	4.24	0.93*
Volatility														
Beta	0.41	0.15	-0.02	-0.46	-0.07	-0.47	26.6	-0.02	-1.04	6.88	-0.15	0.09	5.24	0.02
Residual Volatility	1.96	0.71	-0.51	-1.86	-0.29	-2.24	19.4	-0.12	-1.79	3.82	-0.47	-2.18	3.14	-0.69*
Quality														
Profitability	-1.62	-0.65	-0.35	0.38	2.23	3.85	7.09	0.54	1.72	2.26	0.76*	1.10	1.16	0.95*
Earnings Quality	-1.00	-1.45	-0.58	0.71	2.33	3.33	5.65	0.59*	1.33	1.46	0.91*	1.21	0.78	1.55*
Investment Quality	-3.68	-0.65	0.05	0.79	3.50	7.18	10.7	0.67*	2.21	2.09	1.06*	1.15	0.81	1.41*
Leverage	-0.70	0.55	-1.24	0.47	0.93	1.62	8.25	0.20	0.22	1.88	0.12	-0.11	1.11	-0.10
Earnings Variability	1.23	1.22	0.80	-0.34	-2.90	-4.13	12.8	-0.32	-1.30	3.58	-0.36	-0.33	1.26	-0.26
Yield														
Dividend Yield	-0.75	-0.70	-2.23	-0.06	3.72	4.48	13.1	0.34	2.20	2.63	0.83*	1.00	1.23	0.82*
Growth														
Growth	0.35	0.41	-0.62	0.36	-0.50	-0.85	11.4	-0.07	-0.12	2.48	-0.05	0.91	1.24	0.74*
Liquidity														
Liquidity	-0.22	0.52	0.98	0.76	-2.04	-1.83	16.4	-0.11	-0.79	3.35	-0.23	-0.80	2.31	-0.34

*Analysis over period 31/12/94 to 28/2/18. Annualized statistics based on monthly data. Returns gross of transaction costs. *Sharpe ratios statistically significant at the 5% confidence level, after adjusting for selection bias under multiple testing.*

The value factors we examined (book to price, earnings yield, long- term reversal) generated positive information ratios (IR) across all three settings over the period we studied. In fact, IR improved when we account for other influences, suggesting that value strategies have historically performed better when hedging other factors. On the other hand, size factors performed reasonably well in the simple long-short quintile setting; however, performance deteriorated when we account for other factors. Our analysis confirms the strong historical performance of momentum reported in other empirical studies. IR remained high across all methods, suggesting that momentum strategies have worked well historically irrespective of hedging policy on other factors. The analysis of volatility factors shows that betting against beta has only produced small gains historically in simple settings while low residual volatility performance has been consistently positive across different approaches.

Four of the five quality factors we examined (profitability, earnings quality, investment quality, low earnings variability) had positive excess returns historically across all three-factor return estimation methods while low leverage only had positive excess return in a multivariate setting. The Yield factor had positive excess return across all methods. Growth only produced positive IR in a multivariate regression, suggesting that growth strategies historically have performed better when hedging other exposures. Finally, the liquidity factor experienced negative excess returns over our testing period, confirming the low liquidity premium reported in other empirical studies.

Another challenge in evaluating the results of strategy backtests reported in the literature is the lack of information regarding the number of tests conducted and the potential impact of these multiple tests on the statistical significance of the reported results. In the Appendix, we use a new template proposed by Fabozzi and Prado (2018) to assess the potential impact of selection bias under multiple testing (SBuMT). We find that the relevant Sharpe ratio cutoff point for the backtests reported in Exhibit 1 is 0.57 at the 5% level of significance. By using this cutoff, we see that the Sharpe ratios of 10 of the 16 factors constructed through multivariate regression were significant after adjusting for selection bias.

Using a consistent point-in-time global data set and different factor return estimation methods, we broadly confirmed the existence of positive excess returns associated with the main equity factors reported in the literature. But can investors capture these excess returns in practice? How would the introduction of factors affect the performance and characteristics of different investment strategies? Would the introduction of factor tilts reduce the investability and diversification benefits of index strategies? Can factors enhance active strategies without impairing the manager's ability to select stocks and generate alpha? In the next sections, we use the Black-Litterman framework to show how factors can be integrated in standard market indexes and in actual long-only discretionary portfolios.

INTEGRATING FACTORS IN MARKET INDEXES

Black and Litterman (1992) introduced a general framework for combining market information and active investor views in a consistent manner to construct global fixed-income and multi-asset class portfolios. In a subsequent study, He and Litterman (1999) showed that an unconstrained optimal portfolio that combines market views and investor views can be written simply as the sum of market portfolio weights and the weights of different view portfolios.

Jones, Lim and Zangari (2007) investigated how the Black-Litterman framework can be applied in the management of quantitative strategies. In this setting, equilibrium returns are combined with view portfolios that are based on quantitative factors. They discuss various methods for constructing view portfolios based on factor information. We use two approaches similar to Jones, Lim and Zangari to introduce factor information into an index.

In the first approach, the index represents the market view portfolio that we modify by incorporating factor information captured in factor view portfolios:

$$\boldsymbol{b}^* = \boldsymbol{b} + c \cdot \boldsymbol{P}'\boldsymbol{w} \quad (1)$$

where $\boldsymbol{b}^*$ is a vector containing modified index weights, $\boldsymbol{b}$ is a vector containing initial index weights, $\boldsymbol{P}$ is a matrix with rows containing the weights of factor view portfolios, $\mathbf{w}$ is a vector of weights on the factor view portfolios and c is a scaling parameter. In the second approach, we tilt the index towards factors, to ensure we do not remove any index constituents and we avoid short positions:

$$\boldsymbol{b}^* = \boldsymbol{b} + c \cdot diag(\boldsymbol{b})\boldsymbol{P}'\boldsymbol{w} \quad (2)$$

The next challenge is to translate factor information into view portfolios $\boldsymbol{P}$. We can use univariate and multivariate cross sectional regression to derive factor view portfolio weights. In the univariate regression case, asset returns comprise factor and specific components:

$$\boldsymbol{r} = \boldsymbol{x}f + \boldsymbol{e} \qquad (3)$$

where $\boldsymbol{r}$ is a vector containing asset returns, $\boldsymbol{x}$ is a vector containing asset exposures to the single factor, f is the target factor return and $\boldsymbol{e}$ is a vector of specific returns. In this case, the weights of the factor view portfolio are simply the security exposures to the target factor, scaled by a constant:

$$f = (\boldsymbol{x}'\boldsymbol{x})^{-1}\,\boldsymbol{x}'\boldsymbol{r} = k\boldsymbol{x}'\boldsymbol{r} = \boldsymbol{p}'\boldsymbol{r} \quad (4)$$

In the multivariate case, asset returns are driven by multiple factors as well as specific return sources

$$\boldsymbol{r} = \boldsymbol{X}\boldsymbol{f} + \boldsymbol{e} \qquad (5)$$

where $\boldsymbol{X}$ is now a matrix containing asset exposures to the multiple factors and $\boldsymbol{f}$ is a vector of factor returns. In this case, the factor view portfolio weights are the weights of pure factor portfolios that have unit exposure to the target factor, zero exposure to all other factors and minimum specific risk:

$$\boldsymbol{f} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\,\boldsymbol{X}'\boldsymbol{r} = \boldsymbol{P}\boldsymbol{r} \quad (6)$$

Pure factor portfolios are difficult to implement in practice as they typically contain a large number of holdings, have both long and short positions and experience high portfolio turnover. Melas et al. (2010) explore different methods for implementing factor portfolios with fewer holdings and limited turnover. The factor integration methods we investigate in this study do not require pure factor portfolios to be replicated; these portfolios are only used as input to reweight broad market indexes.

We apply these methods to reweight the MSCI ACWI IMI index and three main regional indexes. We use factor exposure data from MSCI's Global Equity Model for Long-Term Investors to derive the factor portfolio weights. We assign equal weights to the factors, winsorize factor exposures at three standard deviations, limit factor portfolio weights at ±3% and set parameter c at levels that result in active risk of approximately 50 bps. We test four methods of integrating factors into market indexes:

1. Add method (formula 1), using view portfolio weights based on factor exposures (formula 4)
2. Tilt method (formula 2), using view portfolio weights based on factor exposures (formula 4)
3. Add method (formula 1), using view portfolio weights based on factor portfolios (formula 6)
4. Tilt method (formula 2), using view portfolio weights based on factor portfolios (formula 6)

We use these four methods, to modify the index weights by combining them with factor information and compare the performance and investability characteristics of the parent and modified indexes. Exhibit 2 shows that factor-tilted indexes outperformed the parent cap-weighted indexes in the four regions and the historical period we examined, across both portfolio construction methods (add, tilt) and both factor return estimation methods (univariate, multivariate). The add method (formula 1) in particular achieved superior performance while the tilt method (formula 2) had better investability. Furthermore, we observe that the add method worked particularly well when pure factor portfolio weights were used as the factor view portfolios.

As expected, the factor exposures of the reweighted indexes move towards target factors by small amounts. Even though all methods led to approximately the same active risk, add methods achieved more aggressive factor tilts, better absolute and relative risk-adjusted performance and higher attribution to factors (shaded row in the table). Active return attributed to other sources remained generally low across all methods, confirming that no significant unintended exposures or biases were introduced to the indexes as a result of the reweighting process.

Exhibit 2: Integrating Factors in Market Indexes

	ACWI IMI					North America				
MSCI IMI Indexes	MSCI	Univariate		Multivariate		MSCI	Univariate		Multivariate	
	Index	Add	Tilt	Add	Tilt	Index	Add	Tilt	Add	Tilt
Absolute Performance										
Total Return (%)	6.23	6.65	6.52	7.13	6.50	6.60	6.87	6.85	7.02	6.67
Volatility (%)	15.5	15.2	15.2	15.3	15.6	15.1	14.7	14.7	14.8	15.0
Sharpe Ratio	0.27	0.30	0.29	0.33	0.28	0.30	0.32	0.32	0.33	0.30
Relative Performance										
Active Return (%)		0.42	0.29	0.90	0.27		0.27	0.25	0.42	0.07
Tracking Error (%)		0.53	0.43	0.47	0.54		0.60	0.54	0.49	0.55
Information Ratio		0.78	0.66	1.92	0.51		0.44	0.47	0.86	0.12
Act Factor Exposures										
Value		0.03	0.01	0.05	0.01		0.02	0.01	0.01	0.00
Size		-0.04	-0.03	-0.15	-0.07		-0.02	-0.02	-0.09	-0.02
Momentum		0.01	0.01	0.06	0.04		0.01	0.01	0.05	0.03
Volatility		-0.04	-0.04	-0.05	-0.02		-0.04	-0.04	-0.03	-0.01
Quality		0.02	0.02	0.02	0.01		0.01	0.02	0.00	0.00
Yield		0.04	0.03	0.08	0.05		0.03	0.03	0.06	0.03
Growth		0.01	0.00	0.07	0.04		0.00	-0.01	0.05	0.02
Liquidity		-0.05	-0.02	-0.08	-0.02		-0.03	-0.02	-0.04	-0.02
Active Attribution (%)										
Currencies		-0.01	-0.01	0.02	0.01		0.00	0.00	0.00	0.00
Countries		-0.01	-0.02	0.02	0.00		0.00	-0.01	0.00	0.01
Industries		-0.03	-0.01	0.01	0.01		-0.06	-0.04	0.00	0.00
Factors		0.49	0.32	0.87	0.42		0.37	0.32	0.55	0.25
Specific		-0.02	0.01	-0.02	-0.16		-0.05	-0.01	-0.13	-0.19
Total Active		0.42	0.29	0.90	0.27		0.27	0.25	0.42	0.07
Investment Capacity										
Avg No of Stocks	8894	7979	8883	7608	8883	2982	2749	2971	2598	2971
Effect. No of Stocks	534	544	539	587	516	208	209	209	217	198
MCap Coverage (%)	100	99.2	99.8	97.8	99.8	100	99.4	99.8	98.4	99.8
Avg Own (% Float)	0.02	0.03	0.02	0.04	0.02	0.03	0.04	0.03	0.06	0.03
Max Own (% Float)	0.02	0.38	0.03	0.47	0.03	0.03	0.27	0.04	0.65	0.04
Index Concentration										
Avg Wgt Multiplier	1.00	1.50	1.10	2.10	1.10	1.00	1.30	1.10	1.60	1.00
Max Wgt Multiplier	1.00	97.4	60.2	122	61.3	1.00	80.9	60.3	74.6	57.4
Top 10 Sec Wgts (%)	8.40	8.40	8.50	8.10	9.30	15.7	15.6	15.7	15.4	16.6
Max Index Wgt (%)	1.50	1.50	1.50	1.40	1.60	2.80	2.80	2.80	2.70	2.90
Active Share (%)	0.00	3.50	2.80	12.1	11.0	0.00	2.40	2.70	8.80	6.80
Index Replicability										
Annual Turnover	4.20	6.90	5.90	16.0	14.5	3.30	5.30	5.00	13.0	10.4
Day to Trade 95%	0.20	0.30	0.10	0.40	0.10	0.20	0.40	0.20	0.50	0.20
Days to Trade Max	1.00	7.60	1.00	14.4	1.10	0.50	2.20	0.60	4.20	0.50

	EAFE					Emerging Markets				
MSCI IMI Indexes	MSCI	Univariate		Multivariate		MSCI	Univariate		Multivariate	
	Index	Add	Tilt	Add	Tilt	Index	Add	Tilt	Add	Tilt
Absolute Performance										
Total Return (%)	5.34	5.86	5.81	6.12	5.71	9.70	10.2	10.2	10.6	10.2
Volatility (%)	16.5	16.2	16.2	16.3	16.6	22.2	21.9	21.9	21.9	22.1
Sharpe Ratio	0.20	0.23	0.23	0.25	0.22	0.34	0.37	0.37	0.39	0.36
Relative Performance										
Active Return (%)		0.52	0.47	0.78	0.37		0.54	0.46	0.90	0.45
Tracking Error (%)		0.49	0.56	0.46	0.57		0.54	0.57	0.54	0.59
Information Ratio		1.05	0.84	1.71	0.65		1.01	0.81	1.65	0.76
Act Factor Exposures										
Value		0.03	0.02	0.04	0.03		0.03	0.03	0.05	0.02
Size		-0.06	-0.06	-0.14	-0.11		-0.02	-0.03	-0.06	-0.05
Momentum		0.02	0.02	0.05	0.03		0.02	0.02	0.04	0.03
Volatility		-0.04	-0.06	-0.05	-0.02		-0.03	-0.03	-0.03	-0.01
Quality		0.03	0.03	0.02	0.00		0.02	0.02	0.03	0.01
Yield		0.04	0.03	0.05	0.04		0.04	0.04	0.07	0.05
Growth		0.02	0.02	0.06	0.05		0.02	0.01	0.05	0.04
Liquidity		-0.06	-0.04	-0.09	-0.03		-0.04	-0.03	-0.05	-0.02
Active Attribution (%)										
Currencies		0.01	0.00	-0.01	-0.04		-0.02	-0.03	-0.02	-0.01
Countries		0.03	-0.01	0.05	0.00		-0.01	-0.04	0.03	0.02
Industries		0.01	0.03	0.02	0.00		0.06	0.03	0.05	-0.01
Factors		0.52	0.44	0.72	0.44		0.42	0.40	0.66	0.34
Specific		-0.04	0.01	0.00	-0.03		0.09	0.10	0.17	0.11
Total Active		0.52	0.47	0.78	0.37		0.54	0.46	0.90	0.45
Investment Capacity										
Avg No of Stocks	3596	3259	3562	3278	3562	2316	2203	2306	2159	2306
Effect. No of Stocks	300	307	312	325	331	207	208	214	213	213
MCap Coverage (%)	100	99.1	99.7	98.7	99.7	100	99.1	99.4	98.7	99.4
Avg Own (% Float)	0.06	0.08	0.06	0.10	0.06	0.16	0.18	0.16	0.21	0.17
Max Own (% Float)	0.06	0.87	0.09	0.88	0.09	0.16	1.14	0.21	1.08	0.21
Index Concentration										
Avg Wgt Multiplier	1.00	1.50	1.10	1.80	1.20	1.00	1.10	1.00	1.30	1.10
Max Wgt Multiplier	1.0	14.1	1.60	14.8	1.70	1.0	8.8	1.4	9.8	1.4
Top 10 Sec Wgts (%)	12.1	12.0	12.1	11.8	11.7	16.2	16.2	15.9	16.0	16.0
Max Index Wgt (%)	1.80	1.80	1.80	1.70	1.90	3.30	3.30	3.20	3.30	3.40
Active Share (%)	0.00	4.20	4.60	9.70	11.3	0.00	3.00	3.40	6.90	6.90
Index Replicability										
Annual Turnover	4.20	7.20	7.20	13.0	13.6	10.7	12.9	12.4	16.5	16.0
Day to Trade 95%	0.40	0.50	0.30	0.60	0.40	0.70	1.30	0.70	2.00	0.70
Days to Trade Max	1.40	3.60	1.40	4.60	1.60	7.50	20.2	8.10	31.9	8.10

Analysis from Dec. 31, 1998 to March 31, 2018. Performance statistics are annualized based on monthly gross total return in USD.

While add methods, especially when using pure factor portfolio weights, had superior performance, tilt methods led to better investment capacity as they are anchored to the market cap weights of the parent indexes. Add methods also removed approximately 10% of the holdings of the parent index and had slightly lower market cap coverage and moderately higher average ownership as percentage of company float market capitalization. Finally, add methods required higher turnover and would take longer to trade the index around rebalancing for a certain level of assets under management.

All factor integration methods we investigated improved the risk-adjusted performance of market indexes historically. For investors with low or moderate assets under management that can accept a small deterioration in index capacity and liquidity, the add method that combines index weights and pure factor portfolios was the most efficient way of integrating factor information into an index. This method achieved historical information ratios ranging between 0.9 and 1.9 over our sample period. On the other hand, investors managing large index-tracking portfolios may opt for the tilt method that uses factor exposures to reweight the index. This approach achieved lower albeit still impressive historical performance while leaving index investability characteristics virtually unchanged.

INTEGRATING FACTORS IN DISCRETIONARY STRATEGIES

In the previous sections, we confirmed the existence of long-term factor premia in global equities and examined ways of integrating factor information into market indexes. We found that tilting indexes towards factors improved risk-adjusted performance without reducing liquidity, investability and diversification. In this section, we turn to the question of incorporating factor views into discretionary strategies. In these strategies, portfolio managers may have concerns that adding factor tilts may distort their investment process and impact their ability to generate alpha from stock selection.

To address these concerns and avoid substantial changes to an existing discretionary portfolio, we incorporate factors by reweighting portfolio holdings. This method ensures that all existing securities remain in the portfolio post the integration of factor views, albeit with modified weights. Effectively, through this process we do not add or remove any names from the portfolio. We reweight only the existing securities picked by the manager, to introduce tilts towards rewarded factors. We use two sets of factor-related signals to reweight the portfolio, factor exposures (formula 4) and factor alpha, calculated using each factor's historical information ratio and current forecast risk:

$$\alpha_{i,k} = x_{i,k}\sigma_k\omega_k \qquad (7)$$

where $\alpha_{i,k}$ is the factor alpha of security *i* coming from its exposure to factor *k*, $x_{i,k}$ is the exposure of security *i* to factor *k*, σ_k is the forecast risk of factor *k* and ω_k is the historical factor information ratio. In total, we tested three ways of modifying the weights of a discretionary portfolio using factor data:

1. Add method (formula 1), using view portfolio weights based on factor exposures (formula 4)
2. Tilt method (formula 2), using view portfolio weights based on factor exposures (formula 4)
3. Tilt method (formula 2), using view portfolio weights based on factor alpha (formula 7)

We tested these methods of integrating factors in active portfolios on a universe of 1,182 global and international (global ex US) actively managed mutual funds during the period December 2008 to December 2017. We assess the impact of factor tilts across all active funds but also within groups of funds sorted on historical performance. We reweight the active portfolios in our data set using factor exposure and factor alpha information on a monthly basis. Exhibit 3 shows the historical performance of these active mutual fund portfolios before and after the integration of factor tilts.

The original portfolios (first shaded column in Exhibit 3) achieved average outperformance of 0.73% with information ratio of 0.17. Factor exposures were generally modest, with small positive tilts to quality and momentum and negative tilts to size and yield. Most of the outperformance came from security selection (27 bps) while countries and industries each contributed 20 bps. Stock selection made the highest contribution across all four performance quartiles in the original active portfolios.

Modified portfolios based on adding factor exposures (second shaded column in Exhibit 3) achieved outperformance of 1.48% with IR of 0.35: adding factors improved performance substantially both in absolute and risk adjusted terms. Factor exposures show that the factor profile of these active funds moved towards rewarded factors. Performance attribution confirms that all the added active return came from factor tilts that we introduced to the portfolios. Interestingly, performance attributed to security selection remained virtually unchanged at 27 bps before and 26 bps after the factor tilts. So, tilting the portfolios towards rewarded factors added 75 bps to active return without impacting the specific contribution.

Exhibit 3: Integrating Factors in Discretionary Strategies

Historical Performance of Global Equity Funds	Original Portfolios 5Y Performance Quartiles					Modified Portfolios Add Method, Factor Exposure					Modified Portfolios Tilt Method, Factor Exposure					Modified Portfolios Tilt Method, Factor Alpha				
	Q1	Q2	Q3	Q4	All	Q1	Q2	Q3	Q4	All	Q1	Q2	Q3	Q4	All	Q1	Q2	Q3	Q4	All
Absolute Performance																				
Total Return (%)	3.55	5.47	6.47	9.28	6.19	4.30	6.21	7.29	10.0	6.94	4.49	6.39	7.43	10.1	7.09	4.37	6.18	7.24	10.2	6.99
Volatility (%)	18.8	17.9	18.1	18.5	18.3	18.1	17.2	17.5	17.9	17.7	17.9	17.0	17.3	17.7	17.5	18.5	17.7	17.9	18.3	18.1
Sharpe Ratio	0.26	0.38	0.43	0.57	0.41	0.31	0.43	0.49	0.63	0.47	0.33	0.45	0.50	0.64	0.48	0.31	0.42	0.48	0.63	0.46
Perf rel to Benchmark																				
Active Return (%)	-2.35	-0.04	1.28	4.06	0.73	-1.60	0.69	2.04	4.81	1.48	-1.43	0.85	2.20	4.93	1.63	-1.51	0.71	2.03	4.90	1.53
Tracking Error (%)	5.06	3.95	4.13	5.61	4.68	4.86	3.94	4.19	5.48	4.61	4.87	4.00	4.25	5.44	4.64	4.94	4.14	4.28	5.70	4.76
Information Ratio	-0.46	-0.01	0.35	0.79	0.17	-0.31	0.20	0.56	0.96	0.35	-0.27	0.24	0.59	0.99	0.39	-0.29	0.19	0.53	0.93	0.34
Perf rel to Original Port																				
Active Return (%)						0.75	0.73	0.77	0.75	0.75	0.92	0.89	0.93	0.86	0.90	0.83	0.75	0.76	0.84	0.80
Tracking Error (%)						1.14	1.11	1.11	1.18	1.14	1.39	1.36	1.34	1.41	1.37	1.18	1.15	1.15	1.21	1.17
Information Ratio						0.66	0.66	0.69	0.63	0.66	0.66	0.66	0.69	0.61	0.66	0.71	0.65	0.66	0.69	0.68
Dev from Original Port																				
Avg Active Wgt (%)						0.27	0.26	0.27	0.34	0.28	0.32	0.30	0.31	0.38	0.33	0.31	0.29	0.28	0.35	0.31
Max Active Wgt (%)						0.88	0.83	0.85	1.09	0.91	1.34	1.28	1.30	1.57	1.37	1.31	1.24	1.22	1.48	1.31
Active Share (%)						9.66	9.90	9.95	10.3	9.96	11.3	11.8	11.9	12.0	11.8	10.7	10.9	10.9	10.9	10.9
Portfolio Turnover																				
Total Turnover (%,pa)	71.1	66.3	63.6	63.3	66.1	87.1	84.6	80.5	83.5	84.0	90.1	88.0	85.0	87.0	87.5	111.6	107.8	105.3	106.0	107.7
Incre Turnover (%,pa)						16.1	18.2	16.9	20.2	17.9	19.0	21.7	21.4	23.7	21.4	40.5	41.5	41.7	42.8	41.6
Factor Exposures																				
Value	0.02	0.01	-0.04	-0.13	-0.04	0.03	0.02	-0.03	-0.12	-0.02	0.03	0.02	-0.03	-0.11	-0.02	0.06	0.05	0.00	-0.08	0.01
Size	-0.25	-0.18	-0.23	-0.38	-0.26	-0.31	-0.26	-0.30	-0.45	-0.33	-0.32	-0.26	-0.30	-0.45	-0.33	-0.25	-0.21	-0.26	-0.42	-0.29
Momentum	-0.02	0.03	0.07	0.11	0.05	0.02	0.07	0.10	0.16	0.09	0.03	0.07	0.11	0.16	0.09	0.05	0.10	0.14	0.18	0.12
Volatility	0.07	0.05	0.04	-0.02	0.04	-0.03	-0.06	-0.07	-0.12	-0.07	-0.06	-0.09	-0.09	-0.14	-0.09	0.07	0.06	0.06	-0.01	0.04
Quality	0.03	0.04	0.07	0.10	0.06	0.12	0.14	0.17	0.20	0.16	0.14	0.17	0.19	0.22	0.18	0.08	0.09	0.10	0.15	0.10
Yield	-0.07	-0.09	-0.14	-0.23	-0.13	-0.01	-0.03	-0.07	-0.17	-0.07	0.01	-0.01	-0.06	-0.16	-0.06	-0.04	-0.06	-0.11	-0.20	-0.10
Growth	0.03	0.04	0.07	0.13	0.07	0.05	0.06	0.09	0.15	0.09	0.05	0.06	0.09	0.15	0.09	0.04	0.05	0.08	0.13	0.07
Liquidity	0.05	0.04	0.04	0.01	0.04	0.00	-0.02	-0.01	-0.04	-0.02	-0.01	-0.03	-0.02	-0.05	-0.03	0.05	0.04	0.05	0.01	0.04
Active Attribution (%)																				
Styles	-0.45	0.05	0.28	0.66	0.13	0.21	0.73	0.99	1.35	0.82	0.32	0.85	1.12	1.48	0.94	0.40	0.92	1.10	1.58	1.00
Industries	-0.41	0.00	0.28	0.89	0.19	-0.34	0.10	0.35	0.90	0.25	-0.31	0.14	0.38	0.91	0.28	-0.36	0.05	0.36	0.92	0.24
Countries	-0.28	0.21	0.34	0.59	0.21	-0.24	0.18	0.40	0.64	0.24	-0.24	0.19	0.40	0.63	0.25	-0.30	0.19	0.35	0.58	0.20
Currencies	-0.30	-0.06	-0.01	0.08	-0.07	-0.31	-0.08	-0.03	0.08	-0.08	-0.32	-0.09	-0.03	0.07	-0.09	-0.26	-0.03	0.01	0.13	-0.04
Specific	-0.89	-0.24	0.39	1.84	0.27	-0.91	-0.24	0.33	1.85	0.26	-0.87	-0.23	0.33	1.84	0.26	-0.98	-0.40	0.21	1.70	0.13
Total Active	-2.35	-0.04	1.28	4.06	0.73	-1.60	0.69	2.04	4.81	1.48	-1.43	0.85	2.20	4.93	1.63	-1.51	0.71	2.03	4.90	1.53

Analysis from Dec. 31, 2008 to Dec. 31, 2017 using holdings of 1,182 global and international actively managed mutual funds.

Modified portfolios using factor exposures to tilt the original portfolio weights (third shaded column in Exhibit 3) achieved even better results, adding 90 bps to active returns on average and increasing IR from 0.17 to 0.39. This was achieved through slightly more aggressive tilting of original portfolios to rewarded factors. This approach also left the managers' stock selection contribution unchanged.

Portfolios tilted on factor alpha (see fourth shaded column in Exhibit 3) outperformed by 1.53% on average with IR of 0.34. Using factor alpha to tilt the original portfolios did improve performance roughly in line with the other methods. However, using factor alpha had a negative impact on the specific contribution, which declined from 26 to 13 bps on average. Furthermore, the tilts required higher portfolio turnover to implement, compared to the other methods. Transforming exposures into alphas using formula 7 favors factors that have higher information ratio and higher forecast volatility as factor alpha is proportional to the product of factor IR and factor volatility. Exhibit 1 shows that value and momentum are the two factors that score highly on this measure. Indeed Exhibit 3 confirms that we achieve more aggressive tilts towards these two factors when we use factor alpha.

These results suggest that using exposures to tilt active portfolios may be the preferred approach for managers who wish to maintain their security selection contribution and benefit from factor premia. Indeed, irrespective of manager skill prior to the integration of factors, all four quartiles experienced substantial uplift in performance while their specific return contribution remained unchanged. On the other hand, factor alphas may be preferred inputs for certain managers, for example those who use explicit return forecasts in their process as inputs to optimized portfolio construction, or those who wish to place more emphasis on factors with higher historical information ratio and higher forecast volatility. Using factor alphas to tilt active portfolios resulted in performance benefits but also led to a small reduction in specific returns.

INTEGRATING FACTORS IN DISCRETIONARY STRATEGIES: A PRACTICAL EXAMPLE

In this section, we show an example of how to incorporate factors in a specific active portfolio from our database. For illustration, we select the exposure-based tilt method; the other methods proceed similarly. The original and modified portfolio weights and some of the intermediate calculations are presented in Exhibit 4.

The fund in question had 46 stocks as of Dec. 31, 2017, with 63% invested in the U.S., 13% in the U.K., and 24% in other markets. The fund had the largest sector weights in the information technology, consumer discretionary and health care sectors, whereas it had no exposure to the industrials, utilities or real estate sectors. A quick glance at the list of holdings reveals that the fund held well-known, large-cap stocks. From the second column, we also see that holding weights ranged between 5.7% and 0.2%.

Column 3 shows the exposure of the holdings to factor "alpha," which in this example is the average exposure to the eight factors, normalized across the underlying global stock universe. For example, the largest holding, Verizon Communications, had positive exposure to size, yield and value and negative exposure to growth, which corresponds to a relatively cheap, high-yielding, large-cap stock with below average growth prospects. Overall, the fund has an exposure 0.48 to the alpha signal which shows that it had already taken advantage of tilting towards historically rewarded factors.

The hypothetical factor-tilted portfolio is constructed in a two-step process using Formula 2. In the first step, the original holding weights are multiplied by the appropriate multiplier, and, in the second step, the weights are rescaled to sum to 1. The holding-level multipliers are listed in column 4, and are calculated as follows (cf. Formula 2):

$$multiplier_i = 1 + c * \alpha_i \quad (8)$$

where the scaling coefficient c is set so that the active exposure of the tilted fund relative to the original reaches the target level of 0.2. The multipliers were bounded from above and below by 2 and 0.5. These limitations were imposed to limit turnover due to the tilting process, and improve the investability of the resulting fund. The final multipliers are shown in column 4.

Next, in column 5, the original weights are multiplied by the multiplier, and finally, in column 6, the multiplied weights are rescaled to sum to 1. As a result of this process, the exposure of the fund to the alpha signal increased by 0.16. It slightly falls short of the target active exposure of 0.2 because of the investability limitations imposed on the stock-level multipliers. The result of this process was that we tended to overweight stocks with alpha signal exposure above the average exposure of the fund, and tended to underweight stock with lower factor exposures.

In this section, we presented the details of the reweighting process for one fund at one particular date. In our backtests, we repeated the process for all funds to arrive at the statistics in Exhibit 3.

Exhibit 4: A Step by Step Example of Integrating Factors in a Discretionary Portfolio

Holding Name	Original Weight	Factor Alpha	Multiplier	Weight * Multiplier	Rescaled Weight	Value	Size	Momentum	Volatility	Quality	Yield	Growth	Liquidity
VERIZON COMMUNICATIONS INC	5.7%	0.18	1.11	6.3%	4.9%	0.48	0.93	-0.15	-0.66	-0.43	1.51	-0.97	-0.65
JOHNSON & JOHNSON	5.2%	0.59	1.34	7.0%	5.4%	0.04	0.93	0.28	-1.01	0.65	0.38	-0.40	-1.00
MICROSOFT CORP	4.7%	-0.09	0.95	4.5%	3.5%	-0.41	0.93	0.49	0.16	0.33	0.12	-0.11	-0.68
PFIZER INC	4.6%	0.65	1.38	6.3%	4.9%	0.43	0.93	-0.07	-0.77	0.72	1.11	-0.55	-0.74
WH SMITH PLC	4.2%	1.24	1.72	7.2%	5.6%	-0.67	-1.74	0.58	-1.15	2.38	-0.86	-0.41	0.55
AT&T INC	3.9%	0.52	1.30	5.0%	4.0%	0.77	0.93	-0.51	-0.85	0.02	1.90	-0.56	-0.32
CISCO SYS INC	3.9%	0.27	1.15	4.5%	3.5%	0.42	0.93	0.37	-0.22	0.21	0.77	-0.35	-0.37
INTERNATIONAL BUSINESS MACHS	3.7%	0.16	1.10	4.0%	3.1%	0.76	0.93	-0.66	-0.34	0.61	1.24	-0.89	-0.29
TOTAL	3.4%	0.15	1.08	3.7%	2.9%	0.52	1.06	-0.49	0.10	0.27	1.39	-0.36	0.22
SAMSUNG ELECTRONICS CO LTD	3.2%	0.51	1.30	4.2%	3.3%	0.51	1.74	0.74	0.79	1.22	0.22	0.71	-0.71
MAYR MELNHOF KARTON ORD	3.2%	1.43	1.83	5.9%	4.6%	-0.09	-0.82	0.00	-0.47	1.01	-0.08	-0.18	-2.07
WALGREENS BOOTS ALLIANCE INC	3.2%	-0.13	0.92	2.9%	2.3%	0.49	0.48	-1.13	-0.81	0.17	0.25	0.17	0.39
ANTHEM INC	3.1%	0.62	1.36	4.3%	3.4%	0.36	0.30	1.03	-0.07	1.12	-0.31	0.03	-0.14
GAP INC DEL	3.1%	0.91	1.53	4.8%	3.7%	0.52	-0.94	1.10	0.75	2.21	0.51	-0.44	1.11
CVS HEALTH CORP	3.1%	0.18	1.10	3.4%	2.7%	1.08	0.50	-0.92	-0.64	0.45	0.66	-0.39	0.22
CHEVRON CORP NEW	2.7%	1.04	1.61	4.3%	3.4%	-0.08	0.93	0.08	-0.68	0.44	0.95	1.64	-0.63
KDDI	2.6%	0.24	1.14	2.9%	2.3%	0.44	1.28	-0.32	0.09	0.66	0.80	-0.36	-0.31
APPLE INC	2.6%	0.02	1.01	2.6%	2.0%	0.28	0.93	0.66	-0.06	0.03	-0.15	0.21	-0.17
RCL FOODS LIMITED	2.3%	1.93	2.00	4.6%	3.6%	0.98	-1.98	-0.79	0.25	0.03	-0.26	0.66	-2.84
ORACLE CORPORATION	2.2%	-0.22	0.87	1.9%	1.5%	0.35	0.93	-0.06	-0.06	0.00	-0.11	-0.39	-0.69
CHINA MOBILE LTD	2.0%	0.99	1.58	3.2%	2.5%	0.95	0.96	-0.76	-0.69	2.00	1.12	-1.54	-1.44
YOOX NET-A-PORTER GROUP	1.8%	-1.88	0.50	0.9%	0.7%	-1.31	-1.01	-0.86	1.64	-1.63	-1.86	0.70	1.16
TEGNA	1.6%	0.17	1.10	1.8%	1.4%	1.69	-2.05	-0.83	-0.08	-0.31	0.11	-1.04	0.97
DEBENHAMS PLC	1.6%	1.83	2.00	3.2%	2.5%	3.16	-2.78	-2.37	-0.25	1.03	2.21	-1.22	0.70
GLAXOSMITHKLINE	1.5%	-0.11	0.93	1.4%	1.1%	-0.37	0.92	-1.23	-0.61	0.53	1.35	-0.77	-0.13
KINGFISHER	1.5%	0.75	1.43	2.1%	1.7%	0.94	-0.90	-0.35	-0.76	0.94	-0.27	-0.19	0.85
CONNECT GROUP PLC	1.5%	2.41	2.00	2.9%	2.3%	1.33	-3.04	-1.60	-0.20	0.83	2.21	-0.35	-1.19
PALLINGHT	1.4%	0.22	1.13	1.6%	1.3%	0.76	-2.66	-2.09	0.59	-0.72	-1.57	-0.36	-2.02
GANNETT CO INC	1.4%	1.50	1.87	2.6%	2.0%	0.88	-2.55	0.47	0.04	0.95	2.09	-1.17	0.97
UNITEDHEALTH GROUP INC	1.4%	0.56	1.32	1.8%	1.4%	-0.26	0.93	0.93	-0.43	0.92	-0.25	0.40	-0.69
HENKEL STA	1.4%	0.23	1.13	1.6%	1.2%	-0.45	0.36	-0.60	0.17	0.24	-0.52	-0.34	-2.38
EXXON MOBIL CORP	1.3%	0.58	1.33	1.7%	1.3%	0.08	0.93	-0.48	-1.05	0.49	1.09	-0.11	-0.79
INTEL CORP	1.2%	0.24	1.14	1.4%	1.1%	0.62	0.93	0.25	0.23	1.10	0.33	-0.20	-0.03
PERSIMMON	1.2%	0.62	1.36	1.6%	1.2%	0.43	-0.80	0.61	0.21	0.13	0.70	0.08	1.02
CARS.COM	1.1%	0.17	1.10	1.2%	1.0%	0.55	-2.30	-0.10	-0.23	-0.07	-1.02	0.30	1.38
BED BATH & BEYOND INC	1.1%	-0.14	0.92	1.0%	0.8%	2.84	-2.01	-2.36	0.38	1.71	0.52	-2.17	2.32
HEWLETT PACKARD ENTERPRISE	1.1%	-0.11	0.94	1.0%	0.8%	1.04	-0.48	-0.19	0.42	0.97	0.16	-1.72	0.37
SPIRE HEALTHCARE GROUP PLC	1.0%	-0.27	0.85	0.8%	0.6%	0.37	-2.35	-1.85	-0.25	0.31	-1.24	-0.41	0.93
ADIDAS	0.9%	-0.06	0.96	0.9%	0.7%	-1.12	0.14	-0.08	-0.18	0.84	-0.63	0.74	0.49
HP INC	0.8%	0.25	1.14	0.9%	0.7%	0.80	-0.14	0.40	0.47	0.62	0.49	-0.88	0.10
GAZPROM	0.8%	0.25	1.15	0.9%	0.7%	2.01	0.51	-0.76	-0.57	0.22	0.63	-1.41	0.25
DXC TECHNOLOGY	0.7%	1.00	1.58	1.1%	0.9%	-0.03	-0.35	1.00	-0.10	0.85	-0.58	1.59	0.01
VODAFONE GROUP	0.6%	0.80	1.47	0.8%	0.6%	0.04	0.89	-0.08	-0.50	1.47	1.11	-0.15	-0.05
MICRO FOCUS INT	0.3%	-0.41	0.76	0.3%	0.2%	-0.97	-0.60	0.01	-0.43	-1.14	-0.44	0.54	0.60
BIJOU BRIGITTE	0.3%	2.89	2.00	0.6%	0.5%	0.04	-3.00	-0.88	-1.19	1.75	1.82	-0.55	-1.78
GILEAD SCIENCES INC	0.2%	-0.20	0.88	0.2%	0.2%	1.57	0.71	-0.70	-0.40	-0.07	0.68	-1.17	0.32

CONCLUSION

Asset owners require reference benchmarks to provide broad market coverage and diversification. They also require these benchmarks to have high investment capacity so that funds that track such benchmarks can absorb large allocations. Our analysis showed that using the Black-Litterman framework to integrate factor views into benchmark indexes in the examples discussed above did not reduce their market coverage or diversification characteristics. In fact, we observed that factor-tilted benchmarks became less concentrated as tilts generally effected a modest reallocation away from large-cap securities and into mid- and small-cap constituents.

Index fund managers require reference benchmarks to be liquid, investable and tradable, to enable them to manage large index-tracking portfolios efficiently and with relatively low implementation cost. Our results showed that factor-tilted market indexes in the examples discussed above experienced improved performance historically while remaining highly liquid and investable. The tilt method in particular that anchors the modified index weights to the original market-cap weights had turnover and days-to-trade characteristics that were in line with those of parent cap-weighted indexes.

Discretionary managers use fundamental analysis to select stocks and construct portfolios that seek to outperform the market. Many believe that their unique investment process and expert judgment enables them to generate alpha through judicious security selection. However, many discretionary managers operate in an increasingly difficult business and market environment. From a business perspective, they are often under pressure to reduce costs and improve performance. The market environment has also been challenging as quantitative easing increased correlations and a few large technology stocks dominated the market, which may have contributed to making it difficult to generate alpha from stock selection.

Our analysis confirmed that adding factors to active portfolios in the examples discussed above led to substantial performance uplift historically. Crucially, the portfolio characteristics and the managers' stock selection contribution remained largely unchanged following the introduction of factor tilts. The portfolios held exactly the same securities; no names were added or removed. In addition, modified portfolio weights were highly correlated with the original weights that had been established by the discretionary managers. In summary, our process of integrating factors in active portfolios in the examples discussed above improved performance historically without altering the characteristics of these portfolios and without compromising the managers' ability to deploy their skills and generate alpha from stock selection.

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APPENDIX: SELECTION BIAS UNDER MULTIPLE TESTING

Financial researchers and practitioners have long ignored the effects of multiple testing on the significance of their results. In a world where analyzing vast amounts of data has become much cheaper, conducting multiple backtests with significantly different specifications on a strategy, alpha signal or a regression model has become a daily routine. However, with the profusion of backtests, the likelihood of a false discovery also increases significantly.

To illustrate the seriousness of this problem, Fabozzi and Prado (2018) argue that the expected value of the best Sharpe ratio coming out of 100 independent backtests on a random walk would be around 2.5, despite the fact that clearly no alpha exists in this case. To avoid these types of errors, the usual significance statistics have to be adjusted for the fact that the final results are selected from a potentially large number of independent tests. This is what we set out to do in this appendix following the procedure described in Fabozzi and Prado (2018).

Although the factors used in this paper were taken from MSCI's Global Total Market Equity Model, and so not all of them were initially selected for their excess performance virtues, for the purpose of this statistical analysis, we treat all of them as potential alpha signals. The adjustment carried out below is thus approximate, and should only be taken as an illustration of the process.

The methodology described in Fabozzi and Prado (2018) prescribes three steps to address the problem of selection bias under multiple testing. First, we need to define the family of trials, i.e., the collection of all results among which we selected the published result. In our case, during the building of the model, descriptors were aggregated into factors by simple linear combinations, and all individual and aggregated descriptors were tested separately. This brings the size of the trials to 52 (41 descriptors and 11 style factors with multiple descriptors).

Next, we need to define the number of significantly different experiments conducted, or the family size. In the case of strategy backtests, this is equivalent to the number of clusters such that the intra-cluster correlation is significantly higher than the inter-cluster correlation. In the GEMLT model, descriptors entering the definition of a factor were significantly more correlated among themselves than with other factors or descriptors. So this brings the family size to 16 (16 style factors).

Finally, depending on the significance level, the number of observations, and the family size, the adjusted cut-offs for factor return Sharpe ratios can be calculated. For the derivation, we refer again to Fabozzi and Prado (2018). In the below table we plotted the annualized Sharpe ratio cutoffs, at the 5% significance level for various sample sizes and family sizes. We assumed returns were measured at a monthly frequency, since this was the rebalancing frequency for the factor portfolios presented in the paper. We also assumed normal return distribution, but note that adjustments for skewness and kurtosis are possible, and generally would lead to higher thresholds.

Following this analysis, we find that the Sharpe ratio cutoff relevant for the backtests presented in this paper, assuming 5% significance level, 279 monthly observations and 16 test families is 0.57.

Exhibit A1: Sharpe Ratio Cutoffs at the 5% Significance Level for Various Sample Sizes and Family Sizes, Assuming Normality of Returns

		# of families									
	Annualized Sharpe Ratio	1	2	5	10	25	50	100	250	500	1000
# of months	12	1.83	2.25	2.79	3.21	3.79	4.27	4.80	5.61	6.34	7.24
	24	1.22	1.47	1.78	2.00	2.29	2.50	2.71	2.99	3.20	3.42
	36	0.98	1.18	1.41	1.58	1.79	1.94	2.09	2.28	2.43	2.57
	48	0.84	1.01	1.21	1.35	1.52	1.64	1.76	1.92	2.03	2.14
	60	0.75	0.90	1.07	1.19	1.34	1.45	1.55	1.69	1.78	1.88
	120	0.53	0.63	0.74	0.83	0.93	1.00	1.07	1.15	1.21	1.27
	180	0.43	0.51	0.60	0.67	0.75	0.81	0.86	0.93	0.98	1.03
	240	0.37	0.44	0.52	0.58	0.65	0.70	0.74	0.80	0.84	0.88
	300	0.33	0.39	0.47	0.52	0.58	0.62	0.66	0.72	0.75	0.79

CONTACT US

clientservice@msci.com

AMERICAS

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